

*Using Skew Stickiness Ratio to Fine-tune Options Risk*

## 使用波动度倾斜粘性比微调期权风险

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**Abstract** Vega Margin Interval (VMI) and Delta Margin Interval (DMI) are key inputs to scenario-based margin methodologies (e.g., CME SPAN and OCC TIMS). The current Volatility Scan Range (VSR) VSR methods adopted by clearing houses does not recognize the volatility skew. In this paper, we revised the VSR calculation by segmenting the observations into 4 categories, with co-movement of price and volatility considered. The findings suggest that the current methods can result in an understatement of volatility for options writers and overstatement of volatility for options buyers. The impact of SSR is significant and should not be ignored.

【Vega Margin Interval (VMI) 和 Delta Margin Interval (DMI) 是基于情景分析保证金计算方法（例如 CME SPAN 和 OCC TIMS）的关键输入。目前清算所广泛应用的 VMI 方法，Volatility Scan Range (VSR)，没有考虑到隐含波动率的倾斜。在本文中，我们修改了 VSR 计算方法，考虑到价格和市场波动率的关系，将观察结果分为 4 类。研究结果表明，当前的方法可能会导致期权卖方波动率被低估，而期权买方波动率被高估的情况。进一步讲，现货价格与期权波动率的关系对于保证金的计算方法的影响是明显的，不应该被忽视】

## Introduction

The scenario-based margin methodologies are widely adopted by clearing houses around the globe. Vega Margin Interval (VMI) and Delta Margin Interval (DMI) are key inputs for options and futures. The VMI represents the potential variation of market volatility and is calculated by observing implied volatilities at several moneyness levels over a historical period. The DMI is the potential variation of spot prices and is calculated based on historical price movements.

Scenario based methodologies use on a set of hypothetical scenarios where the DMI and VMI are moved up/down. The trades are repriced under the expected spots and expected implied volatilities and the worst-case losses are required as margin against future price movements. The standard SPAN methodology is to move the price up/down +/-100% of the DMI and VMI.

Volatility Scan Range (VSR), which is the VMI methodology used in the CME SPAN® methodology, is defined as the maximum change reasonably likely to occur for the volatility of each option's underlying price (Peter Fortune (2003) and JSCC 2020).

The SPAN® based methodology uses 16 scenarios where scenarios 11 to 14 represent 4 possible combinations of prices and volatilities moving up or down.

## 前言

基于情景的保证金计算方法被全球清算所广泛采用。Vega Margin Interval (VMI) 和 Delta Margin Interval (DMI) 是基于情景的期权和未来合约保证金计算方法的关键输入。VMI 代表市场波动率的潜在变化，通过观察历史时期内同一期权在多个价值上的（moneyness）的隐含波动率来计算。DMI 代表现货价格的潜在变化，根据历史现货价格变动计算。

基于场景的方法基于一组假设的场景，其中 DMI 和 VMI 向上/向下移动。这些交易在预期的现货和预期的隐含波动率下重新定价，最坏情况下的损失需要作为对未来价格变动的保证金。标准的 SPAN 方法是将价格上调/下调 +/-100% 的 DMI 和 VMI。

在 CME SPAN® 方法中，Volatility Scan Range (VSR)，作为 VMI 的计算方法，被定义为每个期权标的价格波动率可能发生的最大变化。

基于 SPAN® 的方法使用 16 种场景，其中场景 11 到 14 表示价格上涨或下跌的价格和波动率的 4 种可能组合。

As shown in Table 1 below, the +1 in price refers to a full up movement in prices, generally 99th percentile of daily historical movements adjusted for MPOR (i.e.,  $\sigma\sqrt{tz}$ ). The +1 movement in Volatility refers to the full up movement by VSR described above.

The impact on trading varies with scenarios. For example, Scenario 12 represents ‘Price moves Up while Volatility moves Down’ (UD). The scenario is bad for long put and good for short put.

如下表 1 所示，价格的+1 是指价格的全幅上涨，通常是根据保证金风险期调整的每日历史变动的第 99 个百分位（即.,  $\sigma\sqrt{tz}$ ）。波动率中的+1 变动是指上述 VSR 的全幅上涨变动。

对交易的影响因情况而异。例如，场景 12 表示“价格上涨，波动性下降”（UD）。这种情况对多头看跌期权不利，对空头看跌期权有利。

Table 1  
Hypothetical Scenarios and Risk Impact

	Scenario 11	Scenario 12	Scenario 13	Scenario 14
	Price Up & Volatility Up (UU)	Price Up & Volatility Down (UD)	Price Down & Volatility Up (DU)	Price Down & Volatility Down (DD)
Price	+1	+1	-1	-1
Volatility	+1	-1	+1	-1
Impact on Trading	Bad For: Short Call Good For: Long Call	Bad For: Long Put Good For: Short Put	Bad For: Short Put Good For: Long Put	Bad For: Long Call Good For: Short Call

This methodology, however, does not recognize the volatility skew or the asymmetrical and dynamical properties of implied volatility and its relationship with spot prices.

The presence of the skew strongly suggests that applying movements shown in Table 1 could understate or overstate risk from option implied volatilities, which we will show through numerical examples documented in this paper.

然而，这种方法没有认识到隐含波动率的波动率偏斜或不对称的动态特性及其与现货价格的关系。

偏斜的存在强烈表明，应用表 1 中所示的变动可能会低估或高估期权隐含波动性的风险，我们将通过本文记录的数字示例来证明这一点。

Methodology

The implied volatilities for an option depend on the volatility skew which tends to price out-of-the-money options at a higher volatility, which can be modelled by volatility skew models like Heston model (Heston 1993). The Skew Stickiness Ratio (SSR ) was introduced by Bergomi in 2009 to address the relationship between spot and volatility movements. The relationship is expressed in the following equation:

$$R_T = \frac{E[d\hat{\sigma}_F^T d \ln (S)]}{\frac{d\hat{\sigma}_{KT}}{d \ln \ln K} |_F E[(d \ln \ln S)^2]}$$

(1)

Where  $R_T$  is the regression coefficient of  $\Delta\hat{\sigma}_F^T$  (daily increments of the ATM volatility with maturity T) on  $\Delta\ln S$  (daily log returns of the underlying).

Mauro Cesa of Barclays (2021) indicated SSR is a measure of the covariance between the spot price and the ATM forward volatility:

$$SSR = cov(X,Y) = \rho(X,Y) \sigma(X)\sigma(Y)$$

(2)

This paper uses covariance to define SSR (Equation 2 above) and for these purposes will use positive or negative values to separate “Sticky Strike” from “Sticky Delta”.

期权的隐含波动率取决于波动率倾斜，而波动率倾斜往往会使价外期权在较高的波动性下定价（参考赫斯顿模型 [Heston 1993]. SSR 由 Bergomi 于 2009 年提出，旨在解决现货和波动率变动之间的关系。该关系用等式(1)表示，

$$R_T = \frac{E[d\hat{\sigma}_F^T d \ln(S)]}{\frac{d\hat{\sigma}_{KT}}{d \ln \ln K} |_F E[(d \ln \ln S)^2]} \quad (1)$$

其中， $R_T$  是  $\Delta\hat{\sigma}_F^T$ （平值期权波动率的每日增量，给定到期日 T）在  $\Delta \ln S$ （期权标的资产每日对数回报）上的回归系数。

巴克莱银行（Barclays）的 Mauro Cesa（2021）提出，公式（2）SSR 是现货价格与平值期权远期波动率之间协方差的衡量标准。

$$SSR = cov(X, Y) = \rho(X, Y) \sigma(X)\sigma(Y) \quad (2)$$

本文使用协方差来定义 SSR（上面的等式 2），为此将使用正值或负值来区分“粘性行权价格”和“粘性 Delta”。

Instead of considering volatility movements on its own, the study linked volatility movements to movements by using SSR in the underlying prices, segmented into the 4 scenarios.

该研究没有单独考虑波动性变动，而是将波动性变动与基础价格的变动联系起来，分为 4 种情况。

While this paper refers to the scenario-based methodologies widely adopted by clearing houses around the globe, the observations made are applicable to the wider industry.

虽然本文提到了全球清算所广泛采用的基于情景的方法，但所发表的意见适用于更广泛的行业。

## Results and Discussions

The sample consists of 337 US (90 major equities and ETFs) and 247 Canadian (out of 300 TMX equities and ETF's) options covering a variety of sectors. The following historical data between Dec 5, 2011, to Dec 3, 2021, were sourced from Bloomberg L.P.

- Spot Price:  $S_t$
- Implied volatility:  $\sigma_t$  for 1-month, and 12-month based on moneyness of 80%,90%,95%, 97.5%, 100%, 102.5%, 105%, 110%, 120%, respectively.
- The average number of days observed was 1946 with a minimum of 64 days

For the purposes of the covariance analysis, only equities with Significance-F below .05 were considered. This is to ensure that the chance of the null hypothesis (there is no correlation between prices and implied volatility) was very low.

Of the 337 equities, 68% met the significance test and were included in SSR calculation. 87% had negative SSRs, ie. prices were inversely correlated with volatility. The positively correlated equities were either “bearish” ETF's, metals, or pharmaceuticals. Of the 198 equities with Significance F < .05, 85% of sample securities selected had SSR's equal to or less than zero, reflective of strong ‘sticky deltas’ effect.

## 结果和讨论

该样本包括 337 个美国（90 个主要股票和 ETF）和 247 个加拿大（在 300 个 TMX 股票和 ETF 中）期权，涵盖各个行业。以下从 2011 年 12 月 5 日到 2021 年 12 月 3 日期间的历史数据来源于彭博社。

- 现货价格:  $S_t$
- 隐含波动率:  $\sigma_t$  基于 1 个月和 12 个月平价波动度的 80%,90%,95%, 97.5%, 100%, 102.5%, 105%, 110%, 120%
- 平均观察天数 1946 天；最短观察天数 64 天

出于协方差分析的目的，仅考虑显著性 F 低于 .05 的股票。这是为了确保原假设（价格与隐含波动率之间没有相关性）的可能性非常低。

在 337 只股票中，68%符合显著性检验，并被纳入 SSR 计算。87%的人有负面的 SSR，即。价格与波动性成反比。正相关的股票要么是“看跌”的 ETF，要么是金属行业或药品行业。在重要性为 F<0.05 的 198 只股票中，85%的样本证券的 SSR 等于或小于零，反映了强烈的“sticky deltas”效应。

Outlined below are key steps followed for the analysis:

下面概述了分析所遵循的关键步骤。

Step	Description
A	<ul style="list-style-type: none"><li>Download sample data <math>S_t, \sigma_t</math> as specified above. 下载现货价格隐含波动率数据</li><li>Convert downloaded to log daily price <math>LN(S_t/S_{t-1})</math> and implied volatility <math>LN(\sigma_t/\sigma_{t-1})</math> returns 转换成对数回报率</li></ul>
B	<p><b>Calculate VSR:</b></p> <ul style="list-style-type: none"><li>For <b>each</b> equity selected the implied volatility data was averaged across the moneyness indicated above. 对于所选的每种股票，隐含波动率数据均按上述 赢利度取平均值 As an example, for 1 MM SPX Index: <math display="block">\sigma_{SPX\ Index,\ 1\ MM} = (\sigma_{80\%} + \sigma_{90\%} + \sigma_{95\%} + \sigma_{97.5\%} + \sigma_{100\%} + \sigma_{102.5\%} + \sigma_{105\%} + \sigma_{110\%} + \sigma_{120\%})/9</math> <math display="block">\sigma_{SPX\ Index,\ 12\ MM} = (\sigma_{80\%} + \sigma_{90\%} + \sigma_{95\%} + \sigma_{97.5\%} + \sigma_{100\%} + \sigma_{102.5\%} + \sigma_{105\%} + \sigma_{110\%} + \sigma_{120\%})/9</math></li><li>Each equity therefore contained a historical set of spot prices and average implied volatility for 1-month and 1-year, respectively. 因此，每只股票分别包含一组 1 个月和 1 年的历史现货价格和平均隐含波动率。</li><li>The VSR is calculated as the 99<sup>th</sup> percentile of the volatility returns over the entire lookback period for the base case. The data was then sorted into the 4 groups in Table 1. E.g. “UU” consists of all days where both price and volatility movements were positive. The 99<sup>th</sup> percentile was used for positive changes in volatility and the 1<sup>st</sup> percentile for negative changes in volatility. 计算为基本情况的整个回溯期内波动率回报的第 99 个百分位数。然后将数据分类到表 1 中的 4 组。例如，“UU”由价格和波动性变动均为正的所有日期组成。第 99 百分位用于波动性的正变化，第 1 百分位用于波动性的负变化。</li></ul>
C	<p><b>Calculate SSR:</b></p> <ul style="list-style-type: none"><li>Calculate correlation and Skew Stickiness Ratio (SSR) as the covariance between the daily price returns and the daily implied volatility returns for all returns for the equity. 计算相关和倾斜粘性比率（SSR）作为股票所有回报的每日价格回报率和每日隐含波动率回报率之间的协方差。<ul style="list-style-type: none"><li>计算相关性和倾斜粘性比率（SSR）作为股票所有回报率的每日价格回报率和每日隐含波动率回报率之间的协方差。</li></ul></li><li>Calculate Significance-F and use it to filter securities. Results with a Significance F of less than .05 were selected, which suggest that there is a 95% chance that the results are significant. 计算重要性-F 并使用它来过滤证券。选择显著性 F 小于 0.05 的结果，这表明结果显著的可能性为 95%。</li></ul>

The results for the two selected securities, SPX and SDS, are provided for one-month volatilities in Table 3. The SPX measures the value of the 500 largest public companies by market capitalization. SDS – ProShare UltraShort S&P 500 is a fund that is aggressively short the S&P 500, which seeks a return that is 2 times the inverse of the daily performance of the S&P 500. SDS represents the opposite view of S&P 500. Table 3 summarizes the results for the above noted specific securities.

两种选定证券 SPX 和 SDS 的结果，用于一个月的波动率。SPX 按市值衡量 500 家最大上市公司的价值。SDS - ProShare UltraShort S&P 500 是一只激进的做空标准普尔 500 指数的基金，其寻求的回报是标准普尔 500 指数每日表现反向的 2 倍。表 3 总结了上述特定证券的结果。



Table 3  
Summary of 2 Specific Securities

Specific Security	Lookback (Year)	$\rho$	SSR	VSR				
				All <sup>1</sup>	UU <sup>2</sup>	UD	DU	DD
SPX – S&P 500	1	-.85	<0	19.4%	4.7%	-17.3%	30.9%	-5.6%
	2	-.75	<0	30.3%	5.4%	-18.9%	34.9%	-10.0%
	5	-.83	<0	20.2%	6.4%	-17.0%	33.0%	-10.1%
	10	-.75	<0	18.1%	7.2%	-16.0%	29.2%	-8.7%
Average				22.0%	5.9%	-17.3%	32.0%	-8.6%
SDS - ProShares UltraShort S&P 500	1	.72	>=0	28.8%	33.9%	-11.7%	16.1%	-22.8%
	2	.44	>=0	41.2%	45.7%	-40.5%	56.5%	-35.2%
	5	.36	>=0	43.7%	53.0%	-43.0%	44.2%	-42.6%
	10	.27	>=0	38.1%	51.1%	-48.9%	55.3%	-42.6%
Average				38.0%	45.9%	-36.0%	43.0%	-35.8%

Note:  
1. Original Method: no securities were filtered.  
2. UU = Price Up & Volatility Up

Table 3 suggests the following patterns of volatility movements when calculated based on individual scenarios:

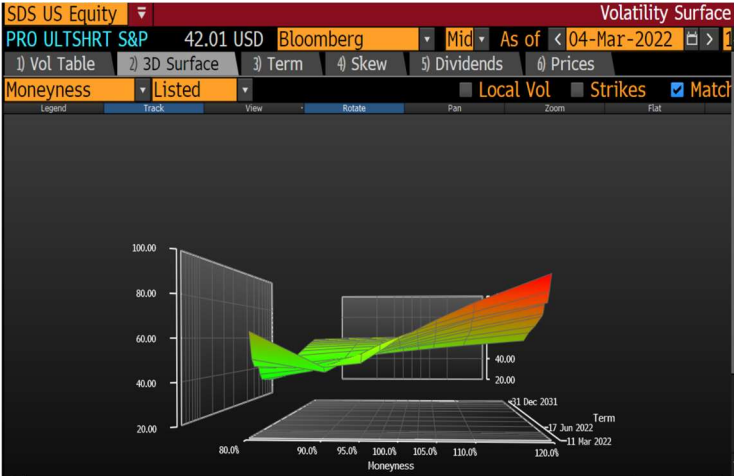
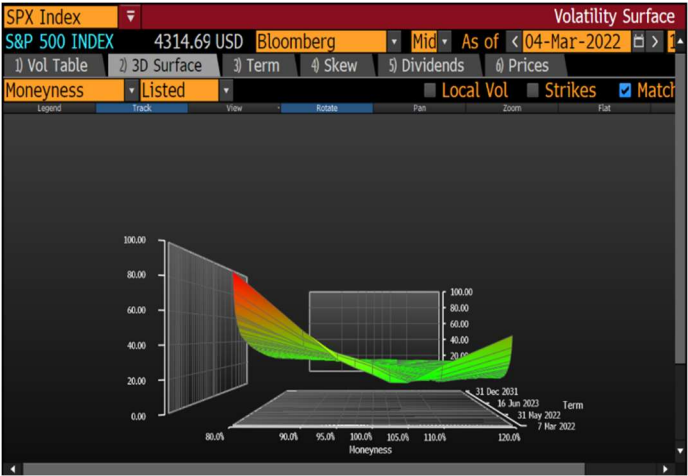
- For SPX under 1-Year Lookback, when spot price is down, market vols may increase another 30.9% as compared with 19.4% suggested by the original method (column All); when spot price is up, market vols may only increase another 4.7% as compared to 19.4% suggested by the original method.
- With SPX, higher up volatility movements occur when prices fall and SSR < 0; With SDS, the opposite is true, that is, higher up volatility movements occur when prices rise for SSR > 0.
- Use of the same VSR across all scenarios is not appropriate.

表 3 根据个别情形计算时，我们有以下波动率变动模式的观察：

- 对于 1 年期回溯下的 SPX，当现货价格下跌时，市场波动率可能会再增加 30.9%，而原始方法建议的为 19.4%（全部列）；当现货价格上涨时，市场波动率可能只会再增加 4.7%，而原始方法建议为 19.4%。
- 对于 SPX，当价格下跌且 SSR<0 时，波动性会更高；对于 SDS，情况恰恰相反，即当 SSR 的价格>0 时，会出现更高的波动率变动。
- 在所有方案中使用相同的 VSR 是不合适的。

The above observations appear consistent to the market performance. As the S&P 500 falls, there is greater demand for SPX put options and conversely greater demand for SDS call options. Local volatility surfaces for the above examples illustrate the skew to the left for the S&P 500 and the skew to the right for the SDS Ultra Short S&P 500. The results for the one-month volatility are shown below.

上述观察结果似乎与市场表现一致。随着标准普尔 500 指数的下跌，对 SPX 看跌期权的需求增加，相反，对 SDS 看涨期权的需求也越来越大。上述示例的局部波动性表面说明了标准普尔 500 指数的左偏斜和标普 500 指数超空头标普 500 指数的右倾斜。一个月波动率的结果如下所示。



- Chart to the left indicates a sticky delta, and to the right a sticky strike.
- Equities in the “Bearish” category had high SSRs -consistent to with market observation when SDS prices rise, S&P shares fall, and volatilities rise.
- Graphs are used with permission of Bloomberg Finance L.P

The impact on Profit/Loss of SPX is shown in Table 4. The risk is the maximum and minimum with short and long, respectively. Short Risk means risk to seller of the option as prices increase, and Long Risk means risk to the holder of an option when the price decreases.

In comparing the P&L’s the put position risk for short is understated by 13.42 % (i.e., ln [-124.79/-109.12]). The long-put P&L is not materially different (i.e. -77.81 vs -76.10). However, call position risks for both short and long are overstated by about 16.8% (i.e., ln (-97.31/-115.15) and 16.47% (i.e., ln (-65.25/-76.94)).

对 SPX 损益的影响见表 4。空头和多头的风险分别趋于最大和最小化。空头风险是指随着价格上涨而对期权卖方的风险，而多头风险是指当价格下跌时期权持有人面临的风险。

在比较损益时，空头看跌头寸风险被低估了 13.42%（即 ln [-124.79/-109.12]）。长看跌损益没有实质性差异（即-77.81 vs -76.10）。然而，空头和多头的看涨头寸风险被夸大了约 16.8%（即 ln（-97.31/-115.15）和 16.47%（即 ln（-65.25/-76.94））。

Table 4  
SPX – S&P 500 Profit/Loss

		Market	Scenario				Risk		P & L	
			UU	UD	DU	DD	Short	Long	Short	Long
	Spot	4,538	4,690	4,690	4,391	4,391				
Original Method	VSR		19.40%	19.40%	19.40%	19.40%				
	Vol	22.50%	27.30%	18.50%	27.30%	18.50%				
	Put	119.02	82.11	41.21	228.14	189.21	228.14	41.21	-109.12	-77.81
	Call	114	230	189	76	37	230	37	-115.15	-76.94
Individual Movement	VSR		4.70%	17.30%	30.90%	5.60%				
	Vol		23.60%	18.90%	30.70%	21.30%				
	Put		64.27	42.92	243.81	200.9	243.81	42.92	-124.79	-76.1
	Call		212	190	92	49	212	49	-97.31	-65.25
Profit/Loss Overstatement /(Understatement)							Put		-13.42%	2.22%
							Call		16.83%	16.47%

Summarized below are the parameters for the calculations:

- Use Black-Scholes option model for the 252-day lookback period and one-month volatilities
- The spot price movements are determined as 3 standard deviations of the historical spot price movement multiplied by the square root of the holding period equal to two days.
- Interest rates for the one-month option is 0.25%, the dividend yield is 1.5% and zero for SPX and SDS, respectively.

Impact on Broader Market is shown by the VSR calculations for all 337 equities per lookback period, with values averaged to illustrate SSR effect for the broader market, as shown in Table 6.

以下是计算的参数：

- 对 Black-Scholes 期权模型中的波动率使用了 252 天和 1 个月的回溯期
- 现货价格变动确定为历史现货价格变动的 3 个标准差乘以等于两天的持有期的平方根。
- 一个月期权的利率为 0.25%，SPX 和 SDS 的股息收益率分别为 1.5%和零。

每个回溯期对所有 337 只股票的 VSR 计算显示了对大盘的影响，其平均值用于说明 SSR 对大盘的影响，如表 6 所示。

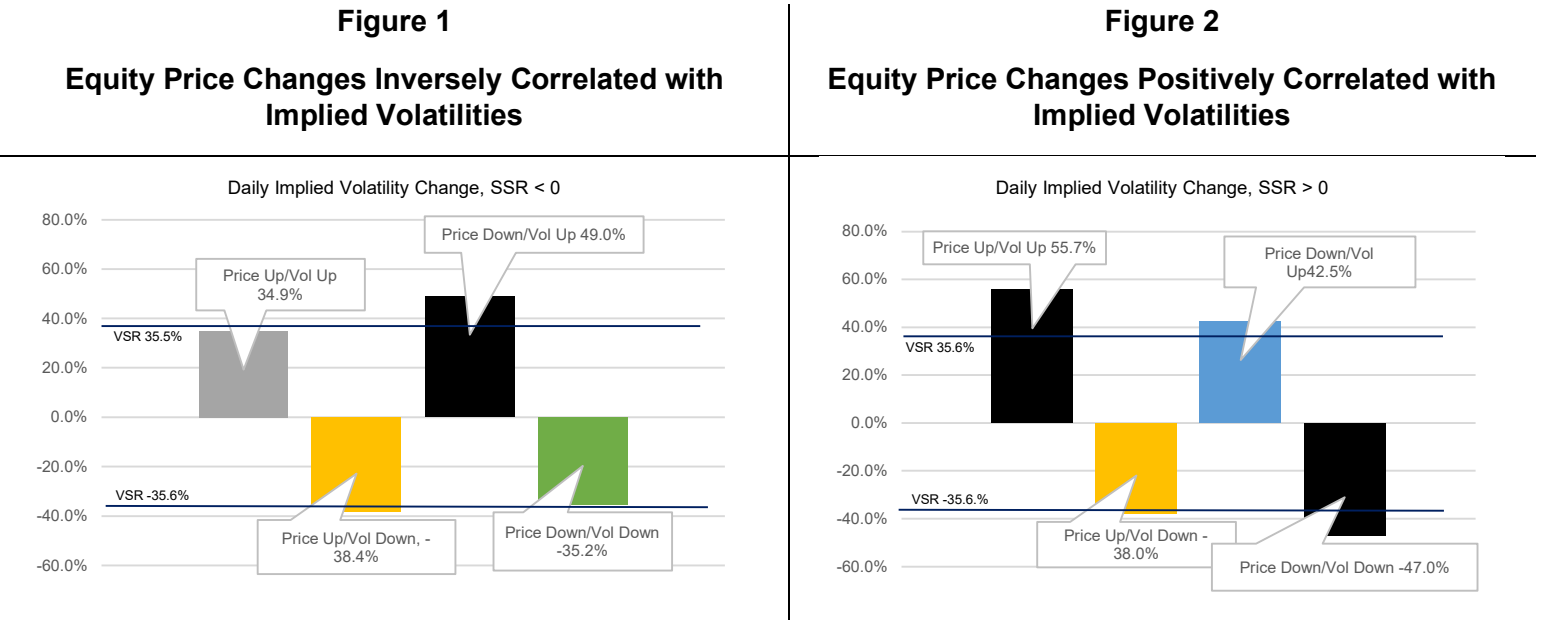
Table 6  
One-Month Volatilities - Average VSRs

Lookback (Year)	VSR Vol Up 337 equities	SSR	No. of Equities	VSR Vol Up	VSR			
					Price Up Vol Up	Price Up Vol Down	Price Down Vol Up	Price Down Vol Down
1	26.8%	< 0	202	18.9%	20.4%	- 22.8%	23.3%	- 18.9 %
		>= 0	29	30.0%	43.7%	- 29.7%	31.5%	- 32.3%
2	35.6%	< 0	262	33.0%	34.9%	- 38.4%	49.0%	- 35.2%
		>= 0	17	38.0%	55.7%	- 38.0%	42.5%	- 47.0%
5	28.5%	< 0	267	24.8%	30.6%	- 32.5%	39.3%	- 32.2%
		>= 0	18	34.0%	47.4%	- 38.8%	41.1%	- 44.6%
10	26.0%	< 0	248	20.7%	26.7%	- 34.2%	30.5%	- 33.4%
		>= 0	13	32.5%	48.1%	- 51.1%	46.8%	- 40.4%

As summarized in Table 7, The impact of the VSR is dependent on the type of risk position, that is, the risk to an option is directional and depends on whether an option is a put or a call, written or purchased. Figure 1 below illustrates the 2-year lookback results (shaded) for equities where SSR is less than zero. The lines represent the VSR Vol for 337 equities. Figure 2 provides the results where SSR is greater than zero.

The risk on short call positions is understated since the worst risk is when prices are up and vols are up at 55.7% which is greater than the 35.6% indicated by the single VSR. In addition, the worst case for calls purchased is when price is down and vols are down, and the call would be valued at 35.6% although the results indicate the value of the call is overstated since vols are down 47%.

如表 7 所示，VSR 的影响取决于风险头寸的类型，即期权的风险是定向的，取决于期权是看跌期权还是看涨期权，是卖出还是买入。下图 1 显示了 SSR 低于零的股票的 2 年回溯期的结果（阴影）。这些线代表 337 只股票的 VSR Vol。图 2 提供了 SSR 大于零的结果。



Conclusions

We revised the approach to the VSR calculation by segmenting the observations into 4 categories: price up and volatility up, price up and volatility down, price down and volatility up, price down and volatility down. We further segmented the results into groups with SSRs < 0 and SSR’s >= 0. We found volatilities were higher than those calculated following the current approach when prices were down and SSR is negative, and when prices were up and SSR is zero or positive.

The findings suggest that the current VSR methods adopted by clearing houses can result in an understatement of volatility for options writers and overstatement of volatility for options buyers. Understatement of volatility for options writers can lead to inadequate margin coverage. For options buyers, an overstatement of volatility can lead to overstatement of the benefits provided by purchased options in a portfolio of options, thereby also leading to inadequate margin coverage.

In managing risks associated with options products, market participants such as market makers, portfolio managers and risk managers, should take into consideration the correlation between price changes and volatility changes in scenario-based methods.

结论

在本文中，我们修改了 VSR 计算方法，将观察结果分为 4 类：价格上涨和波动性上升，价格上涨和波动性下降，价格下跌和波动性上升，价格下跌和波动性下降。我们进一步将结果细分为组：当 SSR < 0 或 SSR 的 >= 0。我们发现，当价格下跌且 SSR 为负时，以及当价格上涨且 SSR 为零或正时，波动率高于按照当前方法计算的波动率。

研究结果表明，清算所目前采用的 VSR 方法可能导致期权卖方波动性被低估，对期权购买者则向反。对期权卖方的波动性低估可能导致保证金覆盖率不足。对于期权购买者而言，对波动性的夸大可能导致在期权组合中夸大购买期权所带来的好处，从而导致保证金覆盖率不足。

在管理与期权产品相关的风险时，市场参与者，如做市商、投资组合经理和风险管理者，应考虑价格变化与基于情景的方法的波动性变化之间的相关性。

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